

A Puzzle of Quantificational Aboutness

Abstract

I will appeal to three plausible assumptions about aboutness to argue against identifying dual quantified propositions, e.g., propositions of the form *Something is F* and of the form *Not everything is non-F*. I will discuss various negative responses to the arguments and argue that most but one aren't of serious concern.

1 Introduction

There is a sense in which universal claims are about all the things they quantify over (call this principle **A**). Thus, for example, the proposition *Everyone will eventually die* is about everyone, *All birds have beaks* is about all birds and *Every proposition is either true or false* is about all propositions. On the other hand, existential claims don't seem to follow this rule; that is, they're not always about everything they quantify over, but only about *some* things—presumably the things that they are true of (**E**). Thus, for instance, *Some men weigh over 200 lbs* is presumably only about those men who weigh over 200 lbs, *Some sets have two members* is about sets with two members and *Some propositions are conjunctive* is about conjunctive propositions. To these add a further plausible, and in fact widely entertained assumption about the interaction of aboutness and negation: a negated proposition is about the same things that its embedded, un-negated proposition is (**N**). Thus both *Bruno isn't sleeping* and *Bruno is sleeping* are about the same things, namely Bruno, and perhaps even sleeping.

These principles together imply that quantificational statements don't express propositions that are identical to their dual propositions; e.g., *something is blue* isn't identical to *not everything is non-blue*. For the latter, due to **N** and **A**, is about everything, whereas the former, due to **E**, isn't; it's only about blue things (and we know that not everything is blue). So the propositions in question don't share the same properties, hence cannot be the same due to Leibniz's law. Similar arguments can be given to the effect that propositions of the form *Everything is blue* and *It's not the case that something is non-blue* are distinct.

In this paper I study these types of arguments and discuss some negative responses which aim at particular premises involved in them, arguing that they're not well-motivated or otherwise appealing. I will then discuss two systematic ways to object the resolutions of the arguments, namely that dual quantificational propositions aren't necessarily identical. Two independent metaphysical views in this regard will be discussed: Ted Sider's logical fundamentalism (Sider 2011),

and Classicism about propositional granularity (Bacon and Dorr MS). It will be argued that the former can be reformulated in the language of higher-order logic in a more appropriate way than Sider’s original formulation, without going against our principles of aboutness, whereas the latter, even though much more serious of a threat, might be countered through abductive reasoning in presence of a larger set of metaphysical considerations.

Before discussing potential responses to the puzzles, some remarks regarding the scope and methodology of the paper are in order. First, much like most of the rest of the literature on aboutness (e.g., Hawke 2018; Perry 1989; Ryle 1933) the main source of the three principles of aboutness introduced above is the intuitions stemming from the ordinary talk of aboutness, as performed in natural languages. That our principles of aboutness in this paper capture such intuitions, although doesn’t necessarily form concrete evidence in their favor, does point at one theoretical merit: explanatory power. That said, other theoretical merits, such as simplicity, strength and uniformity with other related concepts, can and do count, in a wholesome assessment of the principles in question and their consequences. In fact, some such large-scale considerations will be addressed and assessed towards the end of the paper.

Second, the notion of aboutness at play in this paper is concerned with propositions being about objects—concrete or abstract. This makes our conception of aboutness closer to what Hawke (2018) call the ‘subject-predicate’ conception of aboutness—perhaps the earliest conception of aboutness, discussions or use of which going all the way back to Ryle (1933), Putnam (1958), Perry (1989) and N. Goodman (1961). But at least two things distinguish our conception of aboutness from the subject-predicate tradition: (i) I’m *not* committed to any particular subject-predicate account of aboutness, and (ii) I’m *not* concerned about how Boolean connectives, such as conjunction or disjunction, or predications of the form $F(a)$, $G(a, b)$, etc., interact with aboutness. In fact, it is not at all clear that our principles of quantificational aboutness (A and E) should or can be adequately cashed out under such pictures. In general, quantified statements or propositions aren’t standardly considered as predications, and if they are, it is not clear if at all they can be treated similar to non-logical cases of predications when it comes to aboutness.¹ All we have in common with these theories is that a proposition can be about objects. As a result, the type of generic criticisms directed to the typical subject-predicate accounts of aboutness which concern the relationship between

¹In higher-order logic it is in fact customary to treat logical statements as predications (more on this at the end of the paper). For instance the good old universal quantification $\forall x^t \phi$ is often treated as the application instance $\forall^t(\lambda x^t. \phi)$ of the (t -typed) universal quantifier constant \forall^t to the predicate $\lambda x^t. \phi$ of being an x (of type t) such that ϕ , and the conjunctive sentence $\phi \wedge \psi$ as a shorthand for the application $\wedge(\phi, \psi)$ of the conjunction relation \wedge to the sentences ϕ and ψ . In neither of these cases can it be plausibly said that the logical proposition $F(a_1, \dots, a_n)$ is about a_1, \dots, a_n . For example, it’s implausible that *John is sleeping and Gill is abroad*, being the proposition corresponding to $\wedge(\text{John is sleeping}, \text{Gill is abroad})$, is about the proposition *John is sleeping* or the proposition *Gill is abroad*. Similarly, it’s not clear if *Everyone is dying* is about the property of dying, but not any other entity (such as humans). A similar point regarding negation can be made.

aboutness and various types of logical and non-logical predications (as found in, e.g., Hawke 2018) don't in any way affect our work.

2 Rejecting the Premises

We remarked before that in this paper, much like other works in the literature, the main basis upon which our principles of aboutness are found is their intuitive appeal, and that will remain to be our main lead throughout the paper. Of course, at the end of the day, one might insist that (i) our intuitions might be on the wrong track, or (ii) in a larger picture of how things are, more systematic and theoretically robust considerations, e.g., about the metaphysics of propositions, weigh more than our bare intuitions about quantificational aboutness. In what follows, I simply dismiss (i) due to our methodological remarks, but cite some related works across the literature in favor or against our principles.

Aside from their immediate intuitive appeal, it is worth mentioning that there are traces of the implicit or explicit uses of principles like ours, in particular **A**. For instance, Bertrand Russell regularly appeals to this when explaining the nature of quantification and the paradoxes of impredicativity. In particular, Russell was keen on the idea that universal propositions that quantify over propositions are about all propositions, though, due to Vicious Circle reasons, cannot belong to them (hence the ramified hierarchy of propositions). See, for instance, Whitehead and Russell (1910, pp. 39 and 44). Similarly, recently Krämer (2021) has entertained this implicit perspective about aboutness and quantification in his expository work on certain recent puzzles of metaphysical ground—the puzzles of ‘circular aboutness’, as he puts it. The relevant puzzles of ground also have to do with matters of impredicativity, each dealing with the grounds of impredicative propositions that are about themselves (e.g., the proposition $\exists pp$ that there is a true proposition, itself being in the range of quantifier).

Yablo (2014, p. 24), as one of the pioneers of the contemporary study of aboutness also gestures at a similar view when he says ‘*Everything ages* is about everything, me included’, though he doesn’t discuss this in any systematic or extensive way. Putnam (1958) also lays down a special (but now rather outdated) framework, based on ‘amount of information’ in overly simplified first-order languages to argue that universal statements are about all things quantified over (hence endorsing **A**); though in his framework the same holds for existential statements as well (thus rejecting **E**).

As for objections, N. Goodman (1961, p. 5) rejects **A** (or at least that its variant which concerns unrestricted universal quantification) on the grounds that aboutness is like choice, in that, he claims, one can’t choose everything, but only something over another; in a similar way, if a proposition is about **a**, it can’t be about anything else. More generally, Goodman maintains that no statement whatsoever can be about every object, about every class of objects, about every class of classes of objects, etc. But this is dubious in various ways. First, the analogy rests on a wrong assumption about choice. We sure *can* choose indiscriminately unless options are exclusive by nature. Thus in response to ‘Which

dress do you choose to purchase?', one can plausibly respond 'All of them', but to the question 'Which path do you want to take to Phoenix's house?' We can presumably only choose one path (per ride) over the others. Goodman offers no reason to the effect that aboutness in fact *is* supposed to work that way, so we leave this objection here.

Another relevant objection to our puzzles is to deny that aboutness claims are closed under negation, hence rejecting N. Aside from its intuitive import, as we noted before N is almost unanimously endorsed by all accounts of aboutness (see Hawke 2018, for an overview). However, importantly, all of those accounts only deal with aboutness in *propositional* contexts and leave quantification untreated. But it may be argued that even if existential claims are about their true witnesses, *negative* existential claims are not simply about what their embedded existentials are true of; rather, they are about *everything*, because we need everything in order to make negative existentials true.

This argument may sound particularly appealing because we happen to have taken what existential propositions are about to be what makes them true, their witnesses. Now the same view applied to *negated* existentials would imply that they are about everything in the domain because we need them all to make such propositions false. But this generalization builds on a dubious assumption: even though in E the truth of existentials does matter, this constitutes an exception rather than a rule. We didn't limit aboutness to true propositions to begin with. In fact, A and N entail many counterexamples to that. For instance, *Every proposition is true* is false (on pain of contradiction), and yet, by A, it's about every proposition. Similarly, both *Bruno is sleeping* and its negation are arguably about Bruno, but only one of them is true under classical logic.

3 Duality as Identity?

We surveyed some implicit and explicit support for our principle from the literature and discussed some potential objections, which we found rather unappealing. But one might still resist the arguments by rejecting the conclusions on independent grounds: dual propositions just *are* identical, and as a result, at least a premise must be false, no matter how unappealing or counterintuitive that may sound. Two major metaphysical views that can particularly weigh in here are Ted Sider's ideas of logical fundamentality (Sider 2011), and the recently emerged account of relational granularity, Classicism (Bacon and Dorr MS). I'll briefly discuss these below. Since the discussions will depend on a basic understanding of higher-order languages, I'll first give a quick overview of the relevant background on higher-order logic.

In higher-order logic, we can talk about entities of various *types*: individuals (such as people and tables), of type e , propositions, of type $\langle \rangle$ and n -ary relations, of type $\langle t_1, \dots, t_n \rangle$, where t_1, \dots, t_n are types. It is possible and common to treat the logical vocabulary categorically, where the traditional logical statements can be construed as instances of what's called 'application', which is essentially predication (see, e.g., Church 1940; Henkin 1950; Mitchell 1996);

moreover, the operators themselves, and not just the propositions they syncategormatically contribute to form, can be defined in terms of one another using application and ‘abstraction’, the latter being an operation which allows for the formation of certain complex predicates in the language.

For example, if F is a shorthand for the predicate ‘... is fast’, which is of type $\langle e \rangle$, and a for the name ‘Ali’, being of type e , then $F(a)$, the *application* of F to a , translates to the sentence ‘Ali is fast’, of type $\langle \rangle$. Now, consider the sentence ‘Someone loves Jay’, formally represented by $\exists x^e L(x, j)$, with L being a constant of type $\langle e, e \rangle$ standing for the relation of loving, and j of type e a name for Jay. We can create the predicate ‘... is loved by someone’ by *abstracting* from Jay’s name, using lambda abstraction: $\lambda y^e. \exists x^e L(x, y)$. The predicate is taken to stand for the property of being loved by someone. We can similarly create predicates with regards to entities of any arbitrary type t , for any number of arguments.

As for the proof system, in higher-order logic, besides very natural generalizations of the rules of first-order logic (such as Universal Instantiation, Existential Generalization and Modus Ponens) we have two competing principles that govern λ -terms, the second being a weakening of the first (where $[\sigma_1/x_1, \dots, \sigma_n/x_n]\psi$ stands for the simultaneous substitution of the terms σ_i for the variables x_i , for each $i = 1, \dots, n$):

- $(\lambda x_1^{t_1}, \dots, x_n^{t_n}. \psi)(\sigma_1, \dots, \sigma_n) = [\sigma_1/x_1, \dots, \sigma_n/x_n]\psi$, where the type of σ_i is t_i for each $n \geq 1$. $\beta_ =$
- $(\lambda x_1^{t_1}, \dots, x_n^{t_n}. \psi)(\sigma_1, \dots, \sigma_n) \leftrightarrow [\sigma_1/x_1, \dots, \sigma_n/x_n]\psi$, where the type of σ_i is t_i for each $n \geq 1$. β_E

Back to Sider’s logical fundamentalism. According to Sider (2011, Section 10), certain logical connectives and quantifiers are more fundamental than certain others, or as he puts it, ‘carve reality in its joints’. For Sider, the ‘best guide’ for the fundamentality of logical operators, in general, is their indispensability in our fundamental reasoning (see pp. 186 and 216). He particularly pairs conjunction with disjunction, and universal quantifier with existential quantifier, when it comes to comparisons of fundamentality, indicating that only one of them is presumably more fundamental than others.

Next question: *which* logical concepts carve at the joints? I said a moment ago that the sentential connectives of propositional logic carve at the joints. But which ones? Just \wedge and \sim [i.e., \neg]? Just \vee and \sim ? Or perhaps the only joint-carving connective is the Sheffer stroke \uparrow ? Similarly, which quantifier carves at the joints, \forall or \exists ? (*ibid*, p. 217)

Now, the way that Sider discusses these fundamentality precedences in particular the language within which he implements the idea may suggest motivating the identity of dual propositions, that is the troubling identities $\forall x \phi = \neg \exists x \neg \phi$ and $\exists x \phi = \neg \forall x \neg \phi$. To see this, note that Sider (2011) works with first-order languages. Now, assuming (as per the paragraph above) that, e.g., \vee, \neg is more fundamental than \wedge, \neg , this would be naturally cashed out using the identity $\phi \wedge \psi = \neg(\neg \phi \vee \neg \psi)$;

similarly, assuming that \forall is more fundamental than \exists , this can be best captured in first-order language using the propositional identity $\exists x\phi = \neg\forall x\neg\phi$. In short: whichever of \forall and \exists is more fundamental, at least one of the troubling identities of duals emerges.

But as Anonymous (MS[a]) has recently shown, Sider’s views can be rephrased, even more appropriately than in its original first-order formulations, in higher-order languages where expressive resources such as lambda abstraction for predicates are available. The categorematic treatment of the logical vocabulary allows for interdefining them in terms of one another, closer in spirit to Sider’s ideas of logical fundamentality, without invoking the propositional identities that take issue with our principles of aboutness. In particular, we can interdefine conjunction and disjunction, as well as universal and existential quantifiers, in terms of one another without having to endorse the counterpart propositional identities. Here’s how:

- $\wedge = \lambda p^{()}. \lambda q^{()}. \neg(\neg p \vee \neg q)$
- $\vee = \lambda p^{()}. \lambda q^{()}. \neg(\neg p \wedge \neg q)$
- $\forall^t = \lambda X^{(t)}. \neg\exists x^t \neg X(x)$
- $\exists^t = \lambda X^{(t)}. \neg\forall x^t \neg X(x)$

Now, using suitably structured models (as found in, e.g., Benzmüller et al. 2004), it can be shown that under β_E (though not β_+), one can safely endorse any of these relational identities without having to identify the corresponding dual *propositions*, hence make peace between Sider’s idea of logical fundamentalism and our principles of aboutness.²

Another principled approach to motivate the identity of dual propositions, and many more, comes from the newly emerged coarse-grained account of propositions called *Classicism* (Bacon and Dorr MS). Classicism is itself a natural strengthening of another view called *Booleanism* (Dorr 2016), according to which relational entities of any type form a Boolean algebra under the logical connectives of conjunction, disjunction and negation; Classicism adds to these identities that include higher-order identities and quantifiers in a similar way.

More specifically, according to Booleanism, logically equivalent formulas (in classical propositional logic) are identical and form the same relational entities, and according to Classicism, the same holds for a very expressive background higher-order logic. Formally put, Booleanism and Classicism are characterized by the following respective principles (in the case where $n = 0$, by convention we take the λ -terms identical to the sentences embedded in them):

²See `kiani’logicalfundmantallism` for a more comprehensive discussion of higher-order, as well as set-theoretic, formulations of Sider’s ideas and their appropriateness. That paper, in particular, leverages this observation, targeting the duality of conjunction and disjunction instead of universal and existential quantifiers, in the context of grounding and shows that a recent puzzle of ground due to Wilhelm (2020) can, and is quite naturally, avoided within the relevant appropriate higher-order logic in the background.

- $(\lambda x_1^{t_1}, \dots, x_n^{t_n}.\phi) = (\lambda x_1^{t_1}, \dots, x_n^{t_n}.\psi)$ whenever ϕ and ψ are equivalent in classical propositional logic, BOOL
- $(\lambda x_1^{t_1}, \dots, x_n^{t_n}.\phi) = (\lambda x_1^{t_1}, \dots, x_n^{t_n}.\psi)$ whenever ϕ and ψ are equivalent in H , where H is a specific higher-order logic with a strengthening of β_+ in place CLASS

Thus, for example, from Booleanism the following relational and propositional identities follow (where types are omitted for readability): $\lambda p^{()}.q^{()}.p \wedge q = \lambda p^{()}.q^{()}.q \wedge p$, $\lambda p^{()}.q^{()}.p \wedge q = \lambda p^{()}.q^{()}. \neg(\neg p \vee \neg q)$ and $\forall p^{()} \forall q^{()}(p \wedge q = \neg(\neg p \vee \neg q))$. From Classicism all the identities of Booleanism follow. It also follows that $(\lambda y^t z^t. y =^t z) = (\lambda y^t z^t. \forall X^{(t)}(Xy \leftrightarrow Xz))$, $\exists^t(F) = \neg \forall^t(\lambda x^t. \neg F(x))$, $\forall^t(F) = \neg \exists^t(\lambda x^t. \neg F(x))$, $\forall^t = \lambda X^{(t)}. \neg \exists x^t. \neg X(x)$, and $\exists^t = \lambda X^{(t)}. \neg \forall x^t. \neg X(x)$. Thus all the propositional identities that our principles of aboutness purportedly reject, and many more, are independently motivated by Classicism.³

Classicism is an extremely counterintuitive position that storms through a large chunk of established philosophical work in various areas. One is the logic of immediate grounding or ‘because’ (as laid out in, e.g., Fine 2012; Rosen 2010; Schnieder 2011), where a relatively fine-grained account of propositions is needed. For instance, it’s well-known that a conjunctive proposition is grounded in each of its conjuncts, or that a doubly negated proposition is grounded in the proposition that is doubly negated; both of these fail under Classicism, given the standard assumption that grounding is an irreflexive relation. Another context where Classicism contradicts our intuitions is aboutness. Traditionally and intuitively, we take it that the propositions *Snow is white or not white* and *The continuum hypothesis is correct or not* are distinct because one is putatively *about* the continuum hypothesis, whereas the other isn’t (Sider MS). But since both of these propositions are tautologies, they must be identical under Classicism. Moreover, clearly, our principles of quantificational aboutness go against Classicism, as dual quantificational propositions, according to the latter, are identical.

Aside from these blatant counter-examples, both Classicism and Booleanism, and in particular their propositional fragments, have a grand motivation, having to do with paradoxes of grain that their main competitor, Russellian, sentence-like account of propositions, face: the Russell-Myhill paradox (see Dorr 2016; J. Goodman 2017; Hodes 2015; Myhill 1958; Russell 1903; Uzquiano 2015, for various versions of the paradox). In fact Bacon and Dorr bring up this point in response to the counterexamples of the kind mentioned above: ‘While opponents of Booleanism [and Classicism] have pointed to putative counterexamples, they have struggled to provide a comparably systematic and consistent theory which predicts the alleged counterexamples, as opposed to merely accommodating them’ (*ibid.*, p. 3). On a similar note, Sider (MS) says that our primary intuitions

³Notice that although Classicism implies Sider (2011)’s identities, in both propositional and relational forms, the motivating ideas behind the identities here can be quite different: whereas Sider argues for identities of the duals, considerations of non-redundancy about metaphysical priority, if taken up, won’t allow both identities to hold: only *one* of \wedge and \vee , or \forall^t and \exists^t supposedly ‘carves the reality in its joints’.

of aboutness, as illustrated through the example above, need highly structured, Russellian propositions, and hence are subject to the Russell-Myhill paradox of grain, and in fact, that motivates views such as Classicism which have emerged mainly in response to that paradox.

The case of Classicism doesn't seem to be easily dismissed or rephrased in favor of our principles of aboutness: it is too systematic and well-motivated to be ignored. It may indeed be the only position, as of now, that could potentially endanger our principles of aboutness. We leave it open as to whether or not Classicism can be countered in favor of our principles; for now, we may safely assume it's a serious candidate in rejecting our intuitive principles of aboutness.

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