Does Sider's Logical Fundamentalism Endanger Immediate Ground?

Abstract

Wilhelm (2020) has recently shown that certain propositional identities go against widely entertained principles of immediate grounding. This paper discusses one source of motivation for the identities in question, having to do with certain ideas of logical fundamentality due to Sider (2011). It is argued, in particular, that the latter can be rephrased in higher-order logic, in a more appropriate way, in such a way that the troubling propositional identities are avoided.

1 Introduction

Wilhelm (2020) has recently shown that certain propositional identities which he calls 'standard' lead to inconsistencies with widely accepted principles of immediate grounding. Here are the propositional identities in question:

- $\phi \wedge \psi = \neg(\neg\phi \vee \neg\psi)$ $\qquad I_1$
- $\phi \vee \psi = \neg(\neg\phi \wedge \neg\psi)$ $\qquad I_2$

And here are the principles of ground in question (where $\prec$ stands for the relation of grounding):[1]

- $\neg(\phi \prec \phi)$ $\qquad$ IR
- $\phi \prec \neg\neg\psi \leftrightarrow \phi = \psi$ $\qquad$ NG
- $\phi \prec (\psi \wedge \gamma) \leftrightarrow \phi = \psi \vee \phi = \gamma$ $\qquad$ CG
- $\phi \prec (\psi \vee \gamma) \leftrightarrow \phi = \psi \vee \phi = \gamma$ $\qquad$ DG

[1]These are the 'non-factive' version of the principle of ground found in Wilhelm (2020); define ϕ to *factively* grounds ψ (write: $\phi \prec_f \psi$) if $\phi \prec \psi$ and ϕ and ψ are both true. Essentially the same inconsistency results arise here, but we find the non-factive presentation more fundamental and easier to represent. See Fine (2012a, pp. 48-50) and Correia (2017) for some discussions of the factive/non-factive distinction and issues regarding their interdefinability and fundamentality.

- $\phi < \neg(\psi \wedge \gamma) \leftrightarrow \phi = \neg\psi \vee \phi = \neg\gamma$ NCG

- $\phi < \neg(\psi \vee \gamma) \leftrightarrow \phi = \neg\psi \vee \phi = \neg\gamma$ NDG

To see an example, note that by CG we have $\phi < \phi \wedge \phi$, and so by I_1 we have $\phi < \neg(\neg\phi \vee \neg\psi)$. Then from CG and NDG, it then follows that $\phi = \neg\neg\phi$, which goes against IR in presence of NG.

As Wilhelm (2020) mentions, the grounding principles above are widely endorsed across the literature (see, e.g., Correia 2017; Fine 2012b; Krämer 2018). But no reason is offered as to why the propositional identities I_1 and I_2 are 'standard' or should even be endorsed.

This paper discusses one potentially serious motivation for I_1 and I_2 due to Sider (2011), which we call *logical fundamentalism*—the idea that some logical operators are more fundamental than others, or in Sider's special way of putting it, 'carve reality in its joints'. It will be argued that the inconsistency results due to Wilhelm (2020) can be derived under a certain way of cashing out Sider's idea, applied to conjunction and disjunction. It is then shown that Sider's views can be rephrased, more appropriately, within the expressive language of higher-order logic, in such a way that is consistent with the principles of ground outlined above.

I first introduce higher-order logic in a semi-formal way to provide the required technical background. After that I introduce Sider's logical fundamentalism and higher-order formulations of it which avoid the inconsistencies due to Wilhelm. A rigorous presentation of higher-order logic is presented in the technical appendix.

2 Higher-Order Logic

First, some background on higher-order logic (see the technical appendix for a detailed introduction of higher-order logic). We assume there are entities such as individuals, properties of individuals, propositions and polyadic relations that hold for or between these things and more complex entities. We distinguish these entities at the level of syntax by assigning *types* to the relevant terms that stand for them: type e for individuals, $\langle\rangle$ for propositions and $\langle t_1, ..., t_n\rangle$ for n-ary relations ($n \geq 1$) that hold between entities of types t_1, ..., t_n, respectively.

One way to form sentences in higher-order logic is through *application*: for any given type t, if F and a are, respectively, terms of type $\langle t\rangle$ and t, then $F(a)$ is a term of type $\langle\rangle$, i.e., a formula. For example, if F is a shorthand for the predicate '... is funny', and a for the name 'Alex', $F(a)$ translates to the sentence 'Alex if funny'. Similarly, any relational term of type $\langle t_1, ..., t_n\rangle$ can simultaneously apply to entities of types t_1, ..., t_n, respectively, to create sentences. Another way to form terms in higher-order logic is through *abstraction*, which creates predicates out of sentences. For instance, from the sentence 'Someone loves John', formally represented by $\exists x^e L(x, j)$ (with L being a constant of type $\langle e, e\rangle$ standing for the relation of loving, and j of type e a name for John), we can create the predicate '... is loved by someone' by abstracting from the name of John, using lambda abstraction: $\lambda y^e.\exists x^e L(x, y)$. The predicate is taken to stand for the property be being loved by someone.

We can similarly create predicates with regards to entities of any arbitrary type t. Thus the property of being a proposition that has all properties of propositions can be said to be denoted by the predicate $\lambda p^{\langle\rangle}.\forall X^{\langle\langle\rangle\rangle}X(p)$, which itself has type $\langle\langle\rangle\rangle$.

As for the proof system, in higher-order logic, besides very natural generalizations of the rules of first-order logic (such as Universal Instantiation or Existential Generalization—see the appendix) we have two competing principles that govern λ-terms, the second being a weakening of the first (where $[\sigma_1/x_1, ..., \sigma_n/x_n]\psi$ stands for the simultaneous substitution of the terms σ_i for the variables x_i, for each $i = 1, ..., n$):

- $(\lambda x_1^{t_1}, ..., x_n^{t_n}.\psi)(\sigma_1, ..., \sigma_n) = [\sigma_1/x_1, ..., \sigma_n/x_n]\psi$, where the type of σ_i is t_i for each $n \geq 1$. $\beta_=$
- $(\lambda x_1^{t_1}, ..., x_n^{t_n}.\psi)(\sigma_1, ..., \sigma_n) \leftrightarrow [\sigma_1/x_1, ..., \sigma_n/x_n]\psi$, where the type of σ_i is t_i for each $n \geq 1$. β_E

To illustrate, according to $\beta_=$, the proposition that Napoleon was a French emperor is *identical with* the proposition that Napoleon was French and Napoleon was an emperor. Some people have argued against the plausibility of this principle in various contexts (see, e.g., Dorr 2016); in what follows we will offer one such reason in the context of grounding; put formally, $(\lambda x^e.F(x) \wedge E(x))(N) = F(N) \wedge E(N)$. On the other hand, β_E is an extremely plausible, minimal principle that gives us equivalences such as this: Napoleon was a French emperor if and only if Napoleon was French and Napoleon was an emperor: $(\lambda x^e.F(x) \wedge E(x))(N) \leftrightarrow F(N) \wedge E(N)$, with the relevant conventions regarding the constants used in place.

What about logical expressions in our language? When we look at the fine print of many logic books, in particular, introductory ones on propositional and first-order logic, we learn about how logical statements are 'made out of' certain symbols ($\wedge, \vee, \rightarrow, \forall$, etc.) applying to propositional variables and constants, but with no meaning associated to those symbols. We can take a similar approach in higher-order logic. According to the *syncategorematic* approach to the logical vocabulary, they are only linguistic devices that contribute to forming other expressions which denote things in reality; the symbols themselves don't pick up anything out there. Thus, for instance, $\rightarrow$ doesn't denote anything; it merely contributes to the formation of other terms that do, through clauses such as this: whenever ϕ and ψ are terms of type $\langle\rangle$, then so is $\phi \rightarrow \psi$.

In higher-order logic, however, an alternative treatment of the logical vocabulary is available—the *categorematic* approach—according to which each of the logical symbols denotes a proper relation out there in reality that relates the propositions denoted by the formulas it relates to each other. Thus one can take implication $\rightarrow$ be a constant of type $\langle\langle\rangle, \langle\rangle\rangle$, and define of $\phi \rightarrow \psi$ as an application instance of the form $\rightarrow (\phi, \psi)$; the good old cases of quantification, of the form $\forall x^t\phi$, are then construed as instances of application, of the form $\forall^t(\lambda x^t.\phi)$.

From a formal perspective, the two attitudes seem on par, but the categorematic approach, which happens to be the dominant approach in the presentation of higher-order logic (see, e.g., Bacon 2018; Church 1940; Dorr 2016; Henkin 1950; Mitchell 1996), seems to be metaphysically more flexible, as it allows for intelligibly theorizing about the nature and the granularity of the logical vocabulary in a way that the alternative approach doesn't. For example, we can intelligibly ask how to define the connective conjunction $\wedge$ and whether or not it, say, it should be taken as a primitive, identified with, e.g., $\lambda p^t q^t \neg(\neg p \vee \neg q)$ or another truth-functionally equivalent relation.

In particular, the categorematic approach allows for either treating all logical vocabulary as primitive constants, or interdefining some of them in terms of some others that are primitively given. In the former case we will have the following typed logical constants: $\wedge$ (conjunction), $\vee$ (disjunction), $\rightarrow$ (implication) and $\leftrightarrow$ (biconditional), all have of type $\langle\langle\rangle,\langle\rangle\rangle$; $\neg$ (negation) has type $\langle\langle\rangle\rangle$; and for any type t, there is a constant for a (higher-order) universal quantifier $\forall^t$, of type $\langle\langle t\rangle\rangle$, existential quantifier $\exists^t$, of type $\langle\langle t\rangle\rangle$ and identity $=^t$ of type $\langle t,t\rangle$. In the case of indefinability, among various possible ways, we can define some of the logical vocabulary in terms of some primitively given ones; e.g., $\bot := \forall^{\langle\langle\rangle\rangle}(\forall^{\langle\rangle})$, $\neg := \lambda p^{\langle\rangle}(p \rightarrow \bot)$, $\wedge := \lambda p^{\langle\rangle} q^{\langle\rangle}.\neg(p \rightarrow \neg q)$ and $\vee := \lambda p^{\langle\rangle} q^{\langle\rangle}.\neg(\neg p \wedge \neg q)$.

The next section introduces Sider's ideas of logical fundamentality and shows that the categorematic treatment of logical vocabulary offers a promising way to reformulate Sider's ideas more appropriately and in such a way that the inconsistency results found in Wilhelm (2020) can be avoided.

3 Logical Fundamentalism

According to what I call *logical fundamentalism*, explored in Sider (2011, Section 10), some logical operators are more fundamental than others, meaning that the latter can be interdefined in terms of, or derived from, the former. Sider's main motivation for logical fundamentalism is that logical connectives and quantifiers are indispensable in our fundamental scientific theories, and indispensable ideology is the 'best guide' to joint-carving (see, e.g., Sider 2011, pp. 188 and 216). He particularly pairs conjunction and disjunction, as well as the universal and existential quantifiers, in his discussions of logical fundamentality, the idea being that in the case of each pair, presumably only one of them carves reality in its joints (see footnote 2 for a proviso, though). He goes on to say that his approach to fundamentality forces him to make a 'hard choice':

> Next question: *which* logical concepts carve at the joints? I said a moment ago that the sentential connectives of propositional logic carve at the joints. But which ones? Just $\wedge$ and ~ [i.e., $\neg$]? Just $\vee$ and ~? Or perhaps the only joint-carving connective is the Sheffer stroke $\uparrow$? Similarly, which quantifier carves at the joints, $\forall$ or $\exists$? (*ibid*, p. 217)

Here I'm not going to cast any doubt on the general idea that some logical operators are more fundamental than some others, however fundamentality is understood. What I will do, however, is to show that even if one is 'forced' to admit this, one might still have good reasons to question *Sider's* particular way of cashing out the idea, where he pairs $\wedge, \neg$ with $\vee, \neg$ and asks which one is more fundamental; this can potentially lead to the identities involved in the inconsistency result due to Wilhelm (2020).[2]

To see this, note that Sider (2011) only works with first-order languages. Now, assuming, e.g., that $\wedge, \neg$ are more fundamental than $\vee, \neg$, how can we express this in first-order languages? This seems naturally cashed out through the propositional identity $\phi \vee \psi = \neg(\neg\phi \wedge \neg\psi)$. (Similarly, in the present setup, the fundamentality of, e.g., $\forall$ over $\exists$ is naturally cashed out in terms of either of the propositional identities $\exists x\phi = \neg\forall x\neg\phi$ or $\forall x\phi = \neg\exists x\neg\phi$.) So, within the boundaries of the language that Sider works with, one could motivate either of the identities I_1 and I_2, and more.

One may alternatively propose to understand Sider's comparison of the fundamentality of $\vee, \neg$ and $\wedge, \neg$ in terms of the *sets* $\{\vee, \neg\}$ and $\{\wedge, \neg\}$. But even if some sets can be said to be more fundamental than some others (e.g., perhaps, the union of some pairwise distinct sets is less fundamental than the unioned sets), these two particular sets don't seem to stand in such a dynamic. In any case, nothing in Sider's arguments for the fundamentality of the logical operators hinges on sets: they are argued to be fundamental because of their indispensability in fundamental philosophical and scientific theories. But it's hard to see how this hinges on *sets* of connectives and quantifiers as being indispensable; we can and often do formulate our principles of logic, used in our philosophical and scientific investigations, using the connectives and quantifiers *themselves*, not sets that contain them.[3]

A third option is to rephrase the matters of logical fundamentality using higher-order resources. This way, we can target the logical operators *themselves*, and intelligibly talk about their fundamentality in a way that is closest in spirit to Sider's informal discussions. This avoids the awkward presence of sets in the question of fundamentality of the logical operators; it also rises above the unwarranted expressive power of first-order logic due to which one has to harbor questions of logical fundamentality of certain relations through the propositions they syncategorematically contribute to form.[4]

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[2]Note that depending on whether or not one maintains a 'non-redundancy' constraint about fundamentality or joint-carving, one may end up ruling out one of the pairs $\wedge, \neg$, and $\vee, \neg$, over the other *or* be an egalitarian about this and accept both as being equally fundamental; see Sider (2011, pp. 218-220) for more on this. That said, in what follows the choice between these two alternatives won't make a difference in our arguments: no matter which of the identities is believed to hold, the relevant inconsistencies can be avoided.

[3]Also, and in any case, even if one decides to stick to sets, it is worth noting that in the neighboring literature on fundamentality, such as particular 'ontological dependence' and 'entity grounding', it is very common to consider sets as 'less fundamental than', 'dependent on' or are 'entity-grounded in' their members (Correia 2008; Fine 1995; Schaffer 2009). So shifting the question of the fundamentality of $\vee, \neg$ and $\wedge, \neg$ to that of $\{\vee, \neg\}$ and $\{\wedge, \neg\}$ doesn't really seem satisfiable, anyway.

[4]Our higher-order reconstruction of Sider's ideas might be taken to face a challenge from the outset: Sider (2011, Section 9.15) is overall dubious of the fundamentality of higher-order quantification (as opposed to the first-order case), and in particular casts doubt on arguments from natural languages in favor of the innocence or the irreducibility of higher-order quantification to first-order resources (as found in, e.g., Boolos 1984; Rayo and Yablo 2001). But he does offer a criterion that, if satisfied, could be taken in favor of their fundamentality: serving an 'important theoretical purpose'; he doubts, however, if any of the major motivations for higher-order reasoning can satisfy that criterion (see, e.g., p. 213). However, since the date Sider's book has been out, many novel and serious theories and discussions of modality, granularity and grounding have been developed, each putting forward certain systematic views on fundamental notions

Here's how we can replace either of the propositional identities I_1 and I_2 with its corresponding relational identity in our higher-order setting:[5]

- $\wedge = \lambda p^{()} q^{()}.\neg(\neg p \vee \neg q)$
- $\vee = \lambda p^{()} q^{()}.\neg(\neg p \wedge \neg q)$

Now, using suitably structured models (see, e.g., Benzmüller et al. 2004, for various such models in higher-order logic), it can be shown that in the presence of the principle β_E (in place of $\beta_=$), one can safely endorse any of these relational identities without having to identify the corresponding dual *propositions* as following from the logic.

Note that each of these operators can be defined in various other ways, and still there will be no way to guarantee the troubling propositional identities; there are models in which they fail while all the grounding principles, as well as the relevant relational identities, hold. For example, one might take take $\rightarrow$ as more fundamental than $\wedge$ and define $\wedge = \lambda p^{()} q^{()}.\neg(p \rightarrow \neg q)$, and keep the identity $\vee = \lambda p^{()} q^{()}.\neg(\neg p \wedge \neg q)$ or just leave $\vee$ as a primitive. In neither of these cases there's a way to ensure that $\phi \wedge \psi$ is $\neg(\neg\phi \vee \neg\psi)$ or $\phi \vee \psi$ is $\neg(\neg\phi \wedge \neg\psi)$, even though each of these pairs are equivalent due to β_E.

We conclude the section with an important remark: note that our arguments above heavily rely on the use of a particular weakening of $\beta_=$, namely β_E. One might ask, however, if there are any independent reasons to reject β in favor of β_E than just to avoid the troubling propositional identities I_1 and I_2. There are. Fine (2012a) and Rosen (2010) introduce a plausible principle regarding the grounds of λ-terms, according to which, the fact that $[c/x]\phi$ grounds the fact that $(\lambda x.\phi)(c)$. For example, the fact $(\lambda x.U(x) \wedge M(x))(k)$ that Kim is an unmarried man (i.e., a bachelor) is grounded in the fact $U(k) \wedge M(k)$ that Kim is unmarried and a man.

In our higher-order language, this can be more generally expressed using the following schematic formula:

- $[\sigma_1/x_1, ..., \sigma_n/x_n]\psi \prec (\lambda x_1^{t_1}, ..., x_n^{t_n}.\psi)(\sigma_1, ..., \sigma_n)$ $\qquad\lambda$G

Now, it is quite easy to see that λG goes against $\beta_=$ and IR. For if $[\sigma_1/x_1, ..., \sigma_n/x_n]\psi = (\lambda x_1^{t_1}, ..., x_n^{t_n}.\psi)(\sigma_1, ..., \sigma_n)$, then λG leads to the instance of self-grounding $[\sigma_1/x_1, ..., \sigma_n/x_n]\psi \prec [\sigma_1/x_1, ..., \sigma_n/x_n]\psi$.

in metaphysics and logic. In general, recent metaphysics has witnessed a surge in the use of higher-order resources in the study of (often a mix of) various traditional views regarding propositional granularity (Bacon and Dorr (Forthcoming); Dorr 2016; Goodman 2017; Hodes 2015), ground (Fritz 2019; Fritz 2020; Fritz 2021; Goodman (Forthcoming)) and modality (Bacon 2018; Williamson 2013). Most of these works (legitimately) claim to serve 'important theoretical purposes' in our philosophical and logical theories based on the same type of criteria that seem to underlie most other fundamental sciences such as mathematics and physics; see Williamson (2007) and Williamson (2016) for more on such criteria. In any case, in what follows I just assume that higher-order logical vocabulary are equally, if not more, eligible candidates for questions of fundamentality than their first-order counterparts.

[5]Similarly, other putative propositional identities of the form $\exists x^t \phi = \neg\forall x^t \neg\phi$ or $\forall x^t \phi = \neg\exists x^t \neg\phi$ with relational identities of the form $\exists^t = \lambda X^{(t)}.\neg\forall x^t \neg X(x)$ and $\forall^t = \lambda X^{(t)}.\neg\exists x^t \neg X(x)$.

So, the logical framework that saves the propositional logic of impure ground (the system $\mathcal{PH}^E$ in the appendix) is in fact motivated by independent neighboring principles of ground and is thus very welcomed in the relevant context. Hence, Sider's ideas of logical fundamentalism are more appropriately captured using a higher-order logical system that is itself independently motivated by other principles of ground, in such a way that the inconsistencies due to Wilhelm are avoided.

4 Conclusion

I argued that even if, as Sider (2011) claims, there are be some fundamentality patterns lurking through the space of logical operators, Sider's *particular* brand of logical fundamentality (or what he seems to advocate, anyway), which can be taken motivate the propositional identities $\phi \wedge \psi = \neg(\neg\phi \vee \neg\psi)$ and $\phi \vee \psi = \neg(\neg\phi \wedge \neg\psi)$, can be avoided: we can implement the relevant ideas of fundamentality more closely and appropriately in a certain well-motivated higher-order logic, using corresponding *relational* identities without having to go against the standard principles of ground.

It remains open as to what other systematic reasons on propositional identities exist to underlie the identities above, and how the ground theorist can afford to respond to them. One major candidate that motivates these identities, and many more, is the recently emerged coarse-grained account of propositions called Booleanism (Bacon 2018; Dorr 2016); another is its extension Classicism (Bacon and Dorr (Forthcoming)). According to these views, propositional identities are similar to identities of Boolean algebras; clearly, the troubling identities I_1 and I_2 follow from this. Even aside from those identities, these Booleanism and CLassism clearly go against the principles of ground outlined earlier in various other ways. For instance, since ϕ and $\phi \wedge \phi$ are logically equivalent, they are the same; but given the instance of CG according to which $\phi < \phi \wedge \phi$, this goes against the irreflexivity principle, IR. Similarly, ϕ and $\neg\neg\phi$ are identical under these views, and so the principle NG can't fully hold due to IR.

The motivations and applications of Booleanism and Classicism are quite strong, independent from and beyond the scope of our principles of grounding, and larger considerations motivating each of these opposing views often go against one another in various ways. We leave investigating these matters for future work.

Appendix: Technical Background

Here we offer a rigorous presentation of higher-order logic that underlies the less-formal discussions in the main body of the paper.

Definition 1 (Types). The set $\mathcal{T}$ of *types* is the smallest set such that: $e \in \mathcal{T}$ and for any types $t_1, ..., t_n$, where $n \geq 0$, $\langle t_1, ..., t_n \rangle \in \mathcal{T}$.

The type e is reserved for *individuals*, and $\langle t_1, ..., t_n \rangle$ for n-ary *relations*; in case where $n = 0$, by convention we take $\langle t_1, ..., t_n \rangle$ to be the type of *propositions*, and represent it by $\langle\rangle$.

We assume that for any $t \in \mathcal{T}$ there's a denumerably infinite set of *variables* VAR^t of type t and a set of typed non-logical *constants* CST^t which contains a constant $<$ of type $\langle\langle\rangle, \langle\rangle\rangle$ which stands for the relation of strict partial immediate ground. For certain types there are also *logical* constants to be introduced below. We reserve CST^t for the set of all constants, logical or non-logical, of type t. We also define the sets of all variables and constants respectively as $\text{VAR} := \bigcup_{t \in \mathcal{T}^r} \text{VAR}^t$ and $\text{CST} := \bigcup_{t \in \mathcal{T}^r} \text{CST}^t$.

Here's the list of our primitive, typed logical constants: $\wedge$ (conjunction), $\vee$ (disjunction), $\rightarrow$ (implication) and $\leftrightarrow$ (biconditional), all have of type $\langle\langle\rangle, \langle\rangle\rangle$; $\neg$ (negation) has type $\langle\langle\rangle\rangle$; and for any type t, there is a constant for a (higher-order) universal quantifier $\forall^t$, of type $\langle\langle t \rangle\rangle$, existential quantifier $\exists^t$, of type $\langle\langle t \rangle\rangle$ and identity $=^t$ of type $\langle t, t \rangle$. After we introduce the set of terms, we will see how logical statements, in particular, quantifiers behave in this setup.

Definition 2 (Language $\mathcal{L}$). The *terms* in $\mathcal{L}$ are recursively defined as follows: **(i)** if $x^t \in \text{VAR}^t$, then x^t is a term of type t; **(ii)** if $c \in \text{CST}^t$, then c is a term of type t; **(iii)** if ϕ is a terms of type $\langle\rangle$ and for $n \geq 1$, the variables $x_1, ..., x_n$ are pairwise distinct, and respectively of types $t_1, ... ,t_n$, then $\lambda x_1^{t_1}, ..., x_n^{t_n}.\phi$ is a term of type $\langle t_1, ..., t_n \rangle$; **(iv)** if τ is a term of type $\langle t_1, ..., t_n \rangle$, where $n \geq 1$, and for each $i \leq n$, σ_i is a term of type t_i, then $\tau(\sigma_1, ..., \sigma_n)$ is a term of type $\langle\rangle$.

We call a term of type $\langle\rangle$ a *formula*, and when it contains no free variables, a *sentence*. We use the letter t with or without subscripts as metavariables for types, lower-case Greek letters $\tau, \sigma, \phi, \psi, ...$ with or without subscripts as metavariables for general terms, and lower-case or capital English letters $x, y, z, p, q, X, Y, Z, P, Q$, with or without subscripts, as metavariables for variables. The notions of *free* and *bound* variables of terms, substitutions of terms for variables, and *being free for a variable*, are defined as usual. We show the set of free variables in a term σ by $FV(\sigma)$. Also, the set of all terms of $\mathcal{L}$ is denoted by TERM.

Now, we propose a proof theory for our language $\mathcal{L}$.

System $\mathcal{PH}$[6]

Axioms:

1. All theorems of propositional logic PL
2. $\Phi[(\lambda x_1^{t_1}, ..., x_n^{t_n}.\psi)(\sigma_1, ..., \sigma_n)] \leftrightarrow \Phi[[\sigma_1/x_1, ..., \sigma_n/x_n]\psi]$, where the type of σ_i is t_i for each $n \geq 0$ β

[6]Note that $\mathcal{PH}$ is almost exactly the same as H in Bacon and Dorr (Forthcoming), with a minor strengthening: in H, UD, ED and Gen are limited only to cases where F is an λ-term, thus resulting in the more familiar case of $\forall^t(\lambda x^t.\phi \rightarrow \psi) \rightarrow (\phi \rightarrow \forall^t(\lambda x^t.\psi))$. This detail won't make a difference in our discussions in what follows, as either of the systems can be used to establish our point, but we find it worthwhile to have the more general form on display here.

3. $\Phi[\lambda x_1^{t_1}, ..., x_n^{t_n}.\sigma(x_1, ..., x_n)] \leftrightarrow \Phi[\sigma]$, where the type of σ_i is t_i, $n \geq 0$ η

4. $\forall^t F \rightarrow F(\sigma)$, where F is of type $\langle t \rangle$, and the type of σ is t UI

5. $F(\sigma) \rightarrow \exists^t F$, where F is of type $\langle t \rangle$, and the type of σ is t EG

6. $\forall^t(\lambda x^t.\ \phi \rightarrow F(x)) \rightarrow (\phi \rightarrow \forall^t F)$, where $x \notin FV(\phi)$ UD

7. $\forall^t(\lambda x^t.\ F(x) \rightarrow \psi) \rightarrow (\exists^t F \rightarrow \psi)$, where $x \notin FV(\psi)$ ED

8. $\sigma = \sigma$ Ref

9. $\sigma = \tau \rightarrow (F(\sigma) \rightarrow F(\tau))$ Lbz

Rules of Inference:

10. If $\vdash \phi$ and $\vdash \phi \rightarrow \psi$, then $\vdash \psi$ MP

11. If $\vdash F(x^t)$, then $\vdash \forall^t(F)$, where F is of type $\langle t \rangle$ Gen

$\Phi[(\lambda x_1^{t_1}, ..., x_n^{t_n}.\psi)(\sigma_1, ..., \sigma_n)]$ means that Φ is a formula containing an occurrence of $(\lambda x_1^{t_1}, ..., x_n^{t_n}.\psi)(\sigma_1, ..., \sigma_n))$;
$\Phi([\sigma_1/x_1, ..., \sigma_n/x_n]\psi)$ stands for the same formula obtained by replacing occurrences of $(\lambda x_1^{t_1}, ..., x_n^{t_n}.\psi)(\sigma_1, ..., \sigma_n)$ with $[\sigma_1/x_1, ..., \sigma_n/x_n]\psi$. A similar convention applies in the case of η. Notice that the following identities follow from $\mathcal{PH}$:

- $(\lambda x_1^{t_1}, ..., x_n^{t_n}.\psi)(\sigma_1, ..., \sigma_n) =^{\langle\rangle} [\sigma_1/x_1, ..., \sigma_n/x_n]\psi$, where the type of σ_i is t_i for each $n \geq 1$. $\beta_=$
- $\lambda x_1^{t_1}, ..., x_n^{t_n}.\sigma(x_1, ..., x_n) =^{\langle\rangle} \sigma$, where the type of σ_i is t_i, $n \geq 1$ $\eta_=$

A well-known, weaker alternative to $\mathcal{PH}$ is can be obtained by weakening β and η to their 'extensional' variants, as follows, resulting in System $\mathcal{PH}^E$:[7]

- $(\lambda x_1^{t_1}, ..., x_n^{t_n}.\psi)(\sigma_1, ..., \sigma_n) \leftrightarrow [\sigma_1/x_1, ..., \sigma_n/x_n]\psi$, where the type of σ_i is t_i for each $n \geq 1$. β_E
- $\lambda x_1^{t_1}, ..., x_n^{t_n}.\sigma(x_1, ..., x_n) \leftrightarrow \sigma$, where the type of σ_i is t_i, $n \geq 1$ η_E

Note that if, as was discussed in Section 2, one decides to interdefine some of the logical terms in terms of one another (in Sider's way or any other way), one can shrink the resulting proofs system, as some of the axioms will be derivable in terms of some others. For instance, if one defines $\exists^t$ in terms of $\forall^t$ in the usual way, one can drop EG and ED from the proof system, as they can be readily proved by other principles.

[7]Again, $\mathcal{PH}^E$ is almost exactly the same as H_0 in Bacon (2018) and Bacon and Dorr (Forthcoming), with the same minor difference as in the case of $\mathcal{PH}$ and H that was mentioned in the previous footnote.

Finally, note that the main result of the paper can be expressed this way: once we adopt the higher-order system $\mathcal{PH}^E$ over $\mathcal{PH}$, which arguably and due to independent reasons is a better system in the context of grounding *anyway* (considering the principle λG), we can capture Sider's ideas of logical fundamentality in a more appropriate than their original, first-order counterparts, and at the same time avoid the troubling identities I_1 and I_2 that Wilhelm (2020) shows go against the widely accepted principles of immediate grounding; relevant model theories for systems like $\mathcal{PH}^E$ can be found in Benzmüller et al. (2004).

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